
Contents

1	The Science of Information	1
----------	---	---

Part I Components of Information Theory

2	Information Measures	7
2.1	Independence and Markov Chains	7
2.2	Shannon's Information Measures	12
2.3	Continuity of Shannon's Information Measures for Fixed Finite Alphabets	18
2.4	Chain Rules	21
2.5	Informational Divergence	23
2.6	The Basic Inequalities	26
2.7	Some Useful Information Inequalities	28
2.8	Fano's Inequality	32
2.9	Maximum Entropy Distributions	36
2.10	Entropy Rate of a Stationary Source	38
	Appendix 2.A: Approximation of Random Variables with Countably Infinite Alphabets by Truncation	41
	Chapter Summary	43
	Problems	45
	Historical Notes	49
3	The I-Measure	51
3.1	Preliminaries	52
3.2	The I -Measure for Two Random Variables	53
3.3	Construction of the I -Measure μ^*	55
3.4	μ^* Can be Negative	59
3.5	Information Diagrams	61
3.6	Examples of Applications	67
	Appendix 3.A: A Variation of the Inclusion-Exclusion Formula	74

Chapter Summary	76
Problems	78
Historical Notes	80
4 Zero-Error Data Compression	81
4.1 The Entropy Bound	82
4.2 Prefix Codes	86
4.2.1 Definition and Existence	86
4.2.2 Huffman Codes	88
4.3 Redundancy of Prefix Codes	93
Chapter Summary	97
Problems	98
Historical Notes	99
5 Weak Typicality	101
5.1 The Weak AEP	101
5.2 The Source Coding Theorem	104
5.3 Efficient Source Coding	106
5.4 The Shannon-McMillan-Breiman Theorem	107
Chapter Summary	110
Problems	110
Historical Notes	112
6 Strong Typicality	113
6.1 Strong AEP	113
6.2 Strong Typicality Versus Weak Typicality	121
6.3 Joint Typicality	122
6.4 An Interpretation of the Basic Inequalities	131
Chapter Summary	131
Problems	132
Historical Notes	134
7 Discrete Memoryless Channels	137
7.1 Definition and Capacity	140
7.2 The Channel Coding Theorem	149
7.3 The Converse	151
7.4 Achievability	157
7.5 A Discussion	164
7.6 Feedback Capacity	166
7.7 Separation of Source and Channel Coding	172
Chapter Summary	175
Problems	176
Historical Notes	181

8	Rate-Distortion Theory	183
	8.1 Single-Letter Distortion Measures	184
	8.2 The Rate-Distortion Function $R(D)$	187
	8.3 The Rate-Distortion Theorem	192
	8.4 The Converse	200
	8.5 Achievability of $R_I(D)$	202
	Chapter Summary	207
	Problems	208
	Historical Notes	209
9	The Blahut-Arimoto Algorithms	211
	9.1 Alternating Optimization	212
	9.2 The Algorithms	214
	9.2.1 Channel Capacity	214
	9.2.2 The Rate-Distortion Function	219
	9.3 Convergence	222
	9.3.1 A Sufficient Condition	222
	9.3.2 Convergence to the Channel Capacity	225
	Chapter Summary	226
	Problems	227
	Historical Notes	228
10	Differential Entropy	229
	10.1 Preliminaries	231
	10.2 Definition	235
	10.3 Joint and Conditional Differential Entropy	238
	10.4 The AEP for Continuous Random Variables	245
	10.5 Informational Divergence	247
	10.6 Maximum Differential Entropy Distributions	248
	Chapter Summary	251
	Problems	254
	Historical Notes	255
11	Continuous-Valued Channels	257
	11.1 Discrete-Time Channels	257
	11.2 The Channel Coding Theorem	260
	11.3 Proof of the Channel Coding Theorem	262
	11.3.1 The Converse	262
	11.3.2 Achievability	265
	11.4 Memoryless Gaussian Channels	270
	11.5 Parallel Gaussian Channels	272
	11.6 Correlated Gaussian Channels	277
	11.7 The Bandlimited White Gaussian Channel	280
	11.8 The Bandlimited Colored Gaussian Channel	287
	11.9 Zero-Mean Gaussian Noise is the Worst Additive Noise	289

XVIII Contents

Chapter Summary	294
Problems	296
Historical Notes	297
12 Markov Structures	299
12.1 Conditional Mutual Independence	300
12.2 Full Conditional Mutual Independence	309
12.3 Markov Random Field	314
12.4 Markov Chain	317
Chapter Summary	319
Problems	320
Historical Notes	321
13 Information Inequalities	323
13.1 The Region Γ_n^*	325
13.2 Information Expressions in Canonical Form	326
13.3 A Geometrical Framework	329
13.3.1 Unconstrained Inequalities	329
13.3.2 Constrained Inequalities	330
13.3.3 Constrained Identities	332
13.4 Equivalence of Constrained Inequalities	333
13.5 The Implication Problem of Conditional Independence	336
Chapter Summary	337
Problems	338
Historical Notes	338
14 Shannon-Type Inequalities	339
14.1 The Elemental Inequalities	339
14.2 A Linear Programming Approach	341
14.2.1 Unconstrained Inequalities	343
14.2.2 Constrained Inequalities and Identities	344
14.3 A Duality	345
14.4 Machine Proving – ITIP	347
14.5 Tackling the Implication Problem	351
14.6 Minimality of the Elemental Inequalities	353
Appendix 14.A: The Basic Inequalities and the Polymatroidal	
Axioms	356
Chapter Summary	357
Problems	358
Historical Notes	360
15 Beyond Shannon-Type Inequalities	361
15.1 Characterizations of Γ_2^* , Γ_3^* , and $\bar{\Gamma}_n^*$	361
15.2 A Non-Shannon-Type Unconstrained Inequality	369
15.3 A Non-Shannon-Type Constrained Inequality	374

15.4 Applications	380
Chapter Summary	383
Problems	383
Historical Notes	385
16 Entropy and Groups	387
16.1 Group Preliminaries	388
16.2 Group-Characterizable Entropy Functions	393
16.3 A Group Characterization of \overline{T}_n^*	398
16.4 Information Inequalities and Group Inequalities	401
Chapter Summary	405
Problems	406
Historical Notes	408

Part II Fundamentals of Network Coding

17 Introduction	411
17.1 The Butterfly Network	412
17.2 Wireless and Satellite Communications	415
17.3 Source Separation	417
Chapter Summary	418
Problems	418
Historical Notes	419
18 The Max-Flow Bound	421
18.1 Point-to-Point Communication Networks	421
18.2 Examples Achieving the Max-Flow Bound	424
18.3 A Class of Network Codes	427
18.4 Proof of the Max-Flow Bound	429
Chapter Summary	431
Problems	431
Historical Notes	434
19 Single-Source Linear Network Coding: Acyclic Networks ..	435
19.1 Acyclic Networks	436
19.2 Linear Network Codes	437
19.3 Desirable Properties of a Linear Network Code	442
19.4 Existence and Construction	449
19.5 Generic Network Codes	460
19.6 Static Network Codes	468
19.7 Random Network Coding: A Case Study	473
19.7.1 How the System Works	474
19.7.2 Model and Analysis	475
Chapter Summary	478

Problems	479
Historical Notes	482
20 Single-Source Linear Network Coding: Cyclic Networks	485
20.1 Delay-Free Cyclic Networks	485
20.2 Convolutional Network Codes	488
20.3 Decoding of Convolutional Network Codes	498
Chapter Summary	503
Problems	504
Historical Notes	504
21 Multi-Source Network Coding	505
21.1 The Max-Flow Bounds	505
21.2 Examples of Application	508
21.2.1 Multilevel Diversity Coding	508
21.2.2 Satellite Communication Network	510
21.3 A Network Code for Acyclic Networks	510
21.4 The Achievable Information Rate Region	512
21.5 Explicit Inner and Outer Bounds	515
21.6 The Converse	516
21.7 Achievability	521
21.7.1 Random Code Construction	524
21.7.2 Performance Analysis	527
Chapter Summary	536
Problems	537
Historical Notes	539
References	541
Index	561